On almost contact metric hypersurfaces in W_4-manifolds

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1. The famous work by Alfred Gray and Luis M. Hervella [1] contains a classification of the almost Hermitian structures on first order differential-geometrical invariants. In accordance to this classification, all the almost Hermitian structures are divided into 16 classes. Analytical criteria for each concrete structure to belong to one or another class have been obtained [1].

The class of W_4-manifolds is one of so-called small Gray–Hervella classes of almost Hermitian manifolds. Some specialists identify this class with the class of locally conformal Kählerian (lcK-) manifolds that is not absolutely correct. In fact, the W_4-class contains all locally conformal Kählerian manifolds, but coincides with the class of lcK-manifolds only for dimension at least six [2]. W_4-manifolds were studied in detail from diverse points of view by such outstanding mathematicians as Alfred Gray (USA), Vadim Feodorovich Kirichenko (Russian Federation) and Izu Vaisman (Israel).

We remark also that the present communication is a continuation of researches of the author in the area of W_4-manifolds (see, for example, [3], [4], [5] and others).

2. As it is known, an almost Hermitian manifold is a $2n$-dimensional manifold M^{2n} with a Riemannian metric $g = \langle \cdot, \cdot \rangle$ and an almost complex structure J. Moreover, the following condition must hold

$$\langle JX, \ JY \rangle = \langle X, \ Y \rangle, \quad X, Y \in \aleph(M^{2n}),$$

where $\aleph(M^{2n})$ is the module of smooth vector fields on M^{2n} [1]. The fundamental form of an almost Hermitian manifold is determined by the relation

$$F(X, \ Y) = \langle X, \ JY \rangle, \quad X, Y \in \aleph(M^{2n}).$$

An almost Hermitian structure belongs to the W_4-class, if

$$\nabla_X \left(F \right) (Y, Z) \ = \ -\frac{1}{2\left(n - 1\right)} \left\{ \langle X, Y \rangle \, \delta F \left(Z \right) - \langle X, Z \rangle \, \delta F \left(Y \right) - \right.$$

$$\left. - \langle X, JY \rangle \, \delta F \left(JZ \right) + \langle X, JZ \rangle \, \delta F \left(JY \right) \right\}, \quad X, Y, Z \in \aleph(M^{2n}),$$

where δ is the codifferentiation operator and ∇ is the Riemannian connection of the metric $g = \langle \cdot, \cdot \rangle$ [1].

3. The main results are the following:

1) The Cartan structural equations of the general type almost contact metric structure on an oriented hypersurface in a W_4-manifold are obtained;

2) The Cartan structural equations of almost contact metric structures on an oriented hypersurface with type number 0, 1 or 2 in a W_4-manifold are selected;

3) A characterization in terms of the type number (Takagi–Kurihara characterization [6]) of some important classes of almost contact metric structures on hypersurfaces in W_4-manifolds is obtained;

4) A criterion of the minimality of such hypersurfaces in the terms of their type number is established;

5) It is proved that 2- and 3-hypersurfaces in W_4-manifolds do not admit almost contact metric structures belonging to any well-studied classes of almost contact metric structures (cosymplectic, nearly cosymplectic, Kenmotsu, Sasaki etc).

Using the above mentioned fact that the class of W_4-manifolds contains all lcK-manifolds, we conclude that the obtained result are also related to almost contact metric structures on oriented hypersurfaces in lcK-manifolds.

References

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